Summary of Music Theory

This document presents a brief summary of music theory. For more detail, see *How to Play the Guitar by Ear (for mathematicians and physicists)* by Joseph George Caldwell, posted at Internet website http://foundationwebsite.org/Guitar.htm.

Tone (physical definition): A sound produced by vibrating a medium at a particular frequency (or "pitch").
Note: A name given to a tone. The terms tone and note are used somewhat interchangeably. Strictly speaking, a tone is a sound, and a note is a descriptor (name, label).
Fundamental frequency and harmonic (from Internet website http://hyperphysics.phy-astr.gsu.edu/hbase/Waves/funhar.html: The lowest resonant frequency of a vibrating object is called its *fundamental frequency*. Most vibrating objects have more than one resonant frequency and those used in musical instruments typically vibrate at harmonics of the fundamental. A *harmonic* is defined as an integer (whole number) multiple of the fundamental frequency. Vibrating strings, open cylindrical air columns, and conical air columns will vibrate at all harmonics of the fundamental. Cylinders with one end closed will vibrate with only odd harmonics of the fundamental. Vibrating membranes typically produce vibrations at harmonics, but also have some resonant frequencies which are not harmonics.
Octave: A range of tones from a tone of a single frequency to a tone of double that frequency.
Scale: A selection of tones over an octave, including the octave end tones. In referring to a scale, the standard convention is to

name it according to the number of tones including only one end tone. For example, a scale having eight tones including both end tones is referred to as a heptatonic scale (Greek cardinal prefix hepta for seven).

Evenly tempered scale: A scale in which the ratios of the frequencies of successive scale tones are constant (or, the frequencies of the scale tones are equally spaced on a logarithmic scale).

(Western) chromatic scale, or dodecatonic scale: A tempered scale of 12 tones (notes) (13, if both of the octave end tones are counted). If the first note of the scale is of frequency 440 cycles per second (Hertz, or hz), then the scale notes are denoted as A A# B C C# D D# E F F# G G# [A], or as A Bb B C Db D Eb E F Gb G Ab [A]. The bracket is used around the last note, because it is double the frequency of the first note. The twelve notes are usually numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (starting with zero to simplify formulas involving frequencies), but sometimes 1 through 12. In music terminology, a physical tone is called a "note." Confusingly, the interval (distance) between two adjacent notes is called (in music terminology) a "semitone," and the interval between two next-adjacent notes is called a "tone."

As discussed in *How to Play the Guitar by Ear*, there are theoretical physical reasons why the 12-tone scale is preferred over all others.

Melody: A sequence of notes that is musically satisfying.

Harmony: A combination of simultaneously sounded notes that has a pleasing sound, and the progression from one such combination to another.

In what follows, discussion will focus first on consideration of melody, and then or harmony.

Seven-tone (heptatonic) scales. Experience has shown that a large majority of interesting and beautiful musical pieces can be implemented using just seven tones of the 12-tone chromatic (dodecatonic) scale. Documentation of music, construction of musical pieces, understanding, teaching and learning of music

are much simplified by grouping musical pieces according to characteristics of these seven-tone scales. The description of all songs simply by recording their notes on the chromatic scale would not take advantage of the characteristics of songs and musical pieces that arise in human culture, and offer no understanding of why some modulated sound frequencies are pleasing to human beings and others are not.

For more discussion of music scales, see the Wikipedia article, *Scale (music)* at https://en.wikipedia.org/wiki/Scale_(music). The standard music scales are as follows (per the Wikipedia article):

Chromatic, or dodecatonic (12 notes per octave)

Octatonic (8 notes per octave): used in jazz and modern classical music

Heptatonic (7 notes per octave): the most common modern Western scale

Hexatonic (6 notes per octave): common in Western folk music

Pentatonic (5 notes per octave): the anhemitonic form (lacking semitones) is common in folk music, especially in Asian music; also known as the "black note" scale

Tetratonic (4 notes), tritonic (3 notes), and ditonic (2 notes): generally limited to prehistoric ("primitive") music

Examples of hexatonic scales include the whole-tone scale, C D E F♯ G♯ A♯ C; the augmented scale, C D♯ E G A♭ B C; the Prometheus scale, C D E F♯ A B♭ C; and the blues scale, C E♭ F G♭ G B♭ C.

This music-theory summary is limited to discussion of heptatonic scales.

If seven tones are distributed (spread) as evenly as possible across the octave, the distances between them form the sequence 2, 2, 1, 2, 2, 2, 1 or any cyclic shift (rotation) of this sequence, which is called a step pattern. The seven cyclic shifts of this pattern are:

2212221

2122212

1222122
2221221
2212212
2122122
1221222.

These seven scales are called *modes*. That is, a mode of a scale is simply another scale having the same step pattern, but starting at a different point. The step patterns of cyclic shifts of a seven-note subset of the full 12-tone chromatic scale differ because the step pattern contains some two-step intervals and some one-step intervals. The full chromatic scale has no modes, since all notes are separated by the same interval, so that a cyclic shift produces a scale with exactly the same step pattern.

A musical piece is centered, or anchored, on a particular tone (note). That is, the piece (usually) starts on that tone, moves over tones of a scale that starts at that tone, keeps returning to that tone, and ends on that tone. This tone is called the tonic center of the piece.

If a musical piece starts on the first tone of one of the seven above modes, the subjective impression (or “sound”) of the piece will differ noticeably, distinctively, from a piece that starts on the first tone of a different mode. The modes have been given names, as follows:

2212221: Ionian mode, or major scale
2122212: Dorian mode (similar to natural minor scale)
1222122: Phrygian mode (similar to natural minor scale)
2221221: Lydian mode (similar to major scale)
2212212: Mixolydian mode (similar to major scale)
2122122: Aeolian mode (natural minor scale)
1221222: Locrian mode (diminished scale).

Songs based on major scales sound bright, happy, cheery, upbeat and unconstrained. Songs based on a minor key sound ominous, gloomy and somber. Songs based on the Dorian mode sound sad, wistful, forlorn and constrained. An example of a song written in the Dorian mode is *Scarborough Fair*. Most popular

songs are based on major scales, some on minor scales, and a few on Dorian mode. The other modes are not much used. The Locrian mode is not used in popular music (since the diminished scale does not sound particularly pleasant).

The preceding modes / scales are the major ones of modern Western music. There are many other scales in addition to the preceding ones. For more information about modes, see the Wikipedia article *Mode (music)* posted at Internet website https://en.wikipedia.org/wiki/Mode_(music).

Here follows some additional discussion the major and minor scales, which are the most-used scales.

Major (diatonic) scale, or key: A 7-tone subscale of the 12-tone chromatic scale, consisting of notes 0, 2, 4, 5, 7, 9 and 11 (excluding the second endpoint, note 12).

These notes correspond to the white keys on a piano, and are sometimes called "white notes." The ratio of the frequency of the i-th note of the Western chromatic scale to the frequency of the first note of the scale is given by the formula (2 raised to the power i/12), or exp[(i/12) ln(2)]. For the major diatonic scale, these ratios are approximately equal to the following fractions, called "diatonic ratios": 1, 9/8, 5/4, 4/3, 3/2, 5/3, 15/8, 2.

The distances between the notes of the scale are 2, 2, 1, 2, 2, 2, 1; this sequence (2212221) is called the scale step pattern. The scale is named after the name of the starting note. If a musical piece is based on a particular mode, and the scale begins on a particular note, then it is said that the piece is written in the *key* of [note name] [mode name]. Examples: Key of C major: C D E F G A B C; key of A major: A B C# D E F# G#; key of A minor: A B C D E F G. The notes of the key of A minor are exactly those of the key of C major. That is, A minor is the natural minor scale of the major scale starting on the note C (that is, of the scale of C major). The key of A minor is said to be the "relative minor key" of the key of C major. The music-score key signatures for the keys of C major and A minor (or any of the seven modes having the major-scale mode starting on C) are identical (i.e., no sharps or

flats). The musical piece simply starts on A for an A-minor piece and on C for a C-major piece (and D for a Dorian-mode piece).
Minor (diatonic) scale, or key: A 7-tone subscale of the 12-tone chromatic scale, consisting of step pattern 2122xxx, where xxx is 122 (natural minor scale, or key-signature minor scale, or descending melodic minor scale), 221 (ascending melodic minor scale) or 131 (harmonic minor scale). The EarMaster website at https://www.earmaster.com/music-theory-online/ch04/chapter-4-4.html presents a good discussion of minor keys and scales.
The preceding discussion focused on melody. We now turn attention to harmony.
Chord: Any two or more notes (of a diatonic scale) sounded together. The simplest chord is a two-note chord. The best-sounding two-note chords are the ones separated by seven intervals (steps) on the chromatic scale, such as the pair C G or the pair F C. The reason for the pleasant sound is that the second harmonic of the lower note is almost identical to the third harmonic of the higher note, and no interference (irregular frequencies) are heard. Notes that are seven steps apart on the chromatic scale are just five letters apart in the note names (such as C and G). For this reason, it is said that they are a “fifth” apart, or that they are “fifths,” or that G is the fifth of C, or that G is a fifth above C, or that C is a fifth below G. More will be said on this later.
Triad: A chord consisting of three alternate notes of a scale. Example: chord CEG of the scale of C.
Major chord: A triad based on a major scale pattern (i.e., scale step pattern 2212221, chord step pattern 047, or notes 1, 5, and 8 of the chromatic scale (if the notes are numbered 1-12). The chord is given the name of the starting note, e.g., Cmaj, or simply C: CEG. The notes of the chord are called the first (or root), third, and fifth. The first and fifth notes have a strong harmonic relationship (the second harmonic of the lower note is (almost exactly) the third harmonic of the higher note), and sound very well together.

Minor chord: A triad based on a harmonic minor scale pattern (i.e., step pattern 2122131, chord step pattern 037, or notes 1, 4 and 8 of the chromatic scale). The chord is given the name of the starting note, plus an “m”, e.g. Am: ACE. The notes of the chord are called the first, minor third, and fifth.
Diminished chord: A triad with step pattern 036, or notes 1, 4 and 7 of the chromatic scale. Notation: Starting note plus “dim” or “o”. Example: Bdim or Bo: BDF. The notes of the chord are called the first, minor third, and diminished fifth.
Seventh chord: A four-note chord, in which the tenth note of the chromatic scale is added to the other three (e.g., first, fifth and eighth, for a major chord). Example: C7 = CEGA.
Basic chords of a key (“diatonic” chords): all seven triads of a scale, denoted by Roman numerals (I, II, III, IV, V, VI, VII) and having names tonic, supertonic, mediant, subdominant, dominant, submediant, leading note. E.g., the key of C has diatonic chords C (CEG), Dm (DFA), Em (EGB), F (FAC), G (GBD), Am (ACE), Bdim (BDF). The leading-note chord (a diminished chord) is rarely used (it does not sound particularly pleasant). Most musical pieces use only a few chords of the key. For a piece in a major key, the usual chords used are just the three major diatonic chords of the key, viz., I, IV, and V (i.e., the tonic, subdominant, and dominant). If only two chords are used, they are usually I and V. If four major chords are used, they often I, IV, V and II (a fifth above V). If a minor chord is included, it is often VI. There are many other chords (e.g., ninth, suspended, augmented) in addition to the diatonic chords, obtained by adding other notes, or by sharping and flatting certain notes. Example: The main chords in the key of C are C, F, and G (i.e., I, IV and V).
Chord substitutions. Since chord I shares two notes with chords VI or III, the latter may occasionally be substituted, for variety. Similarly, IV shares two notes with II or VI, and V shares two notes with III or VII. E.g., Am may sometimes be substituted for C, or Dm for F.

<u>The circle of fifths (cycle of fifths).</u> A note (on a major diatonic scale) that is four notes above another is said to be a “fifth” above (although it is actually just four notes above). For example, G is said to be a fifth above C. As noted above, notes that are a “fifth” apart are closely harmonically related, and sound particularly good when played together. Similarly, chords whose root notes are a fifth apart sound good together. As noted, many songs are based on chords I, IV, and V. Note that chord I is a fifth above chord IV. The following is a listing of successive “fifths,” arrayed in a circle, called the “circle of fifths.” If memorized, it can be used to quickly determine which chords typically go together in a song, e.g., DAE, or DAEB. It can also be used to help remember the number of sharps or flats in a particular key.

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          C
     F          G
  Bb              D
 Eb                 A
  Ab              E
     Db         B
        Gb = F#
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Key	**Sharps or Flats**
Gb	Bb Eb Ab Db Gb Cb
Db	Bb Eb Ab Db Gb
Ab	Bb Eb Ab Db
Eb	Bb Eb Ab
Bb	Bb Eb
F	Bb
C	No sharps or flats
G	F#
D	F# C#
A	F# C# G#
E	F# C# G# D#
B	F# C# G# D# A#

F# F# C# G# D# A# E#

Memory aids for the circle of fifths: Fat Cats Go Down Alleys Eating Bacon; BEAD-Greatest Common Factor.

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